



## ASSESSMENT OF STUDENT KNOWLEDGE INTEGRATION IN LEARNING FRICTION FORCE

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**Abstract.** *A central objective of science education is to foster a profound comprehension of fundamental scientific principles among students. Research has shown that a highly integrated knowledge structure is a key factor in achieving a deep understanding. This research has developed a friction force conceptual framework to model students' different understandings of friction force from a knowledge integration perspective. Utilizing the established conceptual framework, this study devised and implemented an evaluation of friction force among a cohort of 598 grade-10 students in China. The assessment outcomes were then analyzed quantitatively and qualitatively. The results show that the conceptual framework model effectively represents the knowledge structures of students at different levels of knowledge integration, and the students' understanding of friction force can be divided into three levels: novice, intermediate, and expert. Furthermore, emphasizing the passivity of friction force can help students gain a deeper understanding of the concept of friction force, thereby forming a more comprehensive knowledge structure. This study provides an effective strategy for students to progress from novices to experts.*

**Keywords:** *conceptual understanding, knowledge integration, conceptual framework, friction force*

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### Introduction

Scientific education should foster the development of transferable knowledge and skills, enabling students to meet the demands of modern production and daily life (National Research Council, 2012). However, exam-oriented education often prioritizes the memorization of knowledge over its comprehension. Furthermore, problem-solving is frequently reduced to recalling superficial features of problem contexts (Alonso, 1992; Bao & Koenig, 2019). This approach leads to students excelling in solving familiar problems but struggling when faced with novel or unprecedented scenarios (Chiu et al., 2007; Kim & Pak, 2002; King, 1992; Nurrenbern & Pickering, 1987), indicating a superficial understanding of concepts and a lack of profound comprehension (Champagne et al., 1982; Krathwohl, 2002; Nakhleh, 1993; Rivet & Krajcik, 2008). Numerous studies analyzing students' problem-solving behaviors have identified the organization of knowledge as a pivotal differentiator between expert and novice learners (Champagne et al., 1982; Chi et al., 1981; Hardiman et al., 1989; Sabella & Redish, 2007; Snyder, 2000). The variance in students' conceptual understanding can be modeled through the construction, activation, and linking of concepts (Linn, 2006; Liu et al., 2011; Sabella & Redish, 2007).

To enhance students' conceptual understanding and to facilitate targeted instructional guidance, it is essential to assess the level of students' knowledge integration. In recent studies, Bao et al. (2019) have developed a conceptual framework tool to model the cognitive structures and the reasoning pathways during problem-solving of students as they progress from novices to experts. The framework delineates three distinct developmental levels of knowledge integration: novice, intermediate, and expert. Novices' knowledge structures display fragmented features with loosely connected concepts, closely tied to the contexts encountered in textbooks and lectures. Throughout the problem-solving process, novices typically employ strategies that match contextual features with memorized operational procedures and



equations (Chen et al., 2020; Chi et al., 1981; Larkin et al., 1980). Students at an intermediate level progressively cultivate knowledge structures that are partially integrated, which facilitates their capacity to perform logical analyses within contexts that are familiar to them. However, the incomplete integration of knowledge limits their ability to apply existing knowledge to unfamiliar contexts (Bao et al., 2002; Bao & Redish, 2006; Zou et al., 2023). In contrast, experts possess highly integrated knowledge structures centered around the central idea, facilitating the flexible application of knowledge to solve problems across various contexts (Chi et al., 1981). The conceptual framework for student learning is tailored to specific content areas, thus requiring the creation of distinct frameworks for individual conceptual subjects. This model has been successfully developed and implemented across various physical science topics, including, for example, force and motion (Nie et al., 2019), momentum (Xu et al., 2020), Newton's third law (Bao & Fritchman, 2021).

Friction force is a fundamental concept in physics, and integral to the curriculum from elementary to university education. Despite its significance, research in this area remains limited, primarily focusing on students' misconceptions and the challenges they encounter when solving problems (Arons, 1997; Besson et al., 2007; Caldas & Saltiel, 1995; Kızılcık et al., 2021). However, the specific causes of these difficulties remain unknown. Furthermore, this dearth of research on varying levels of comprehension of the friction force impedes the development of tailored pedagogical approaches that could effectively address these educational challenges. To assess students' comprehension of friction force and elucidate the challenges they encounter in understanding this concept, this study aimed to model students' different conceptual understandings of friction force from the lens of knowledge integration. Specifically, the following work was done in this study:

- (1) Construct a conceptual framework for friction force to simulate the knowledge structures of students with varying levels of conceptual understanding.
- (2) Design an evaluative instrument grounded in the conceptual framework to investigate students' comprehension of friction force and infer their knowledge structures.

## Conceptual Framework

### *Friction Force Conceptual Framework*

A conceptual framework is a tool used to describe the knowledge structures of students on a specific topic. The elements within a conceptual framework include not only concepts but also the basic characteristics of problem situations and variables, as well as task goals. The interconnections among the elements reveal the knowledge structures of students with varying levels of conceptual understanding and illuminate their reasoning processes as they apply these elements to problem-solving. This research has formulated a conceptual framework for friction force, acting as a theoretical construct to outline the knowledge structure of students. The framework's components and structure were derived and refined through expert analysis of the physical concepts documented in literature and the learning behaviors of students. Friction force encompasses various types, and the focus of this study was on static friction and sliding friction, which are topics covered in the physics curriculum at the high school level in China.

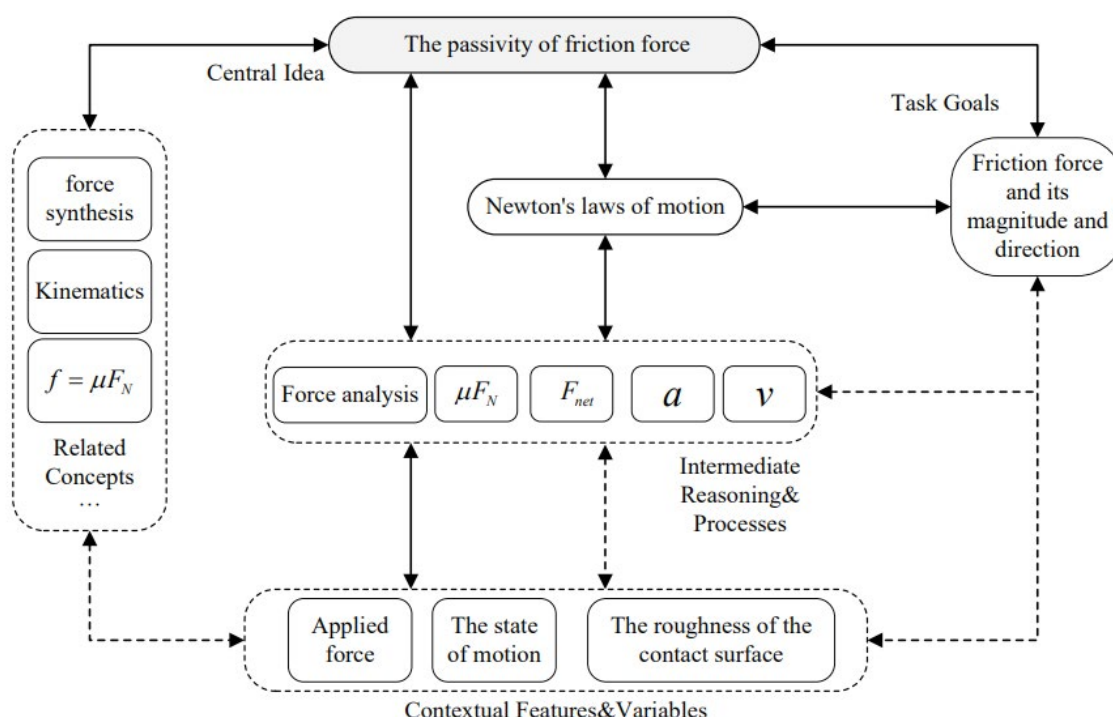
Drawing on prior research, the initial step in constructing a conceptual framework involves identifying a central concept that acts as an anchor, integrating the related concepts (Bao & Fritchman, 2021; Dai et al., 2019; Liu et al., 2022; Nie et al., 2019; Tong et al., 2023; Xie et al., 2021; Xu et al., 2020; Zou et al., 2023). Friction force, a "passive" force (Arons, 1997), exhibits an inherently complex phenomenology that is contingent upon the materials in question and the conditions under which they are applied (Arons, 1997; Besson et al., 2007). This passivity is manifested in three key aspects:

- (1) Conditions for the generation of friction force: firstly, mutual contact and pressing between objects; secondly, the contact surface is rough; and thirdly, the presence of either relative motion or a tendency for it between the objects.
- (2) Direction: The direction of friction force is always opposite to the direction of relative motion or relative motion tendency of the object. The objects here refer to those that can be considered as mass points. When dealing with complex objects, such as the direction of friction force on a car, it is necessary to separately discuss the motion situation of the front and rear wheels relative to the ground.
- (3) Magnitude: The magnitude of friction force is related to the properties of the two contacting surfaces (such as their roughness) and the normal force between them.

According to the passivity of friction force, whether it exists or not, as well as its magnitude and direction, is determined by other external forces and the object's state of motion (e.g. at rest, uniform straight-line motion,

and accelerated motion). When addressing problems, experts take into account the passivity of friction force and analyze its actual effect based on the object's actual state of motion and the forces acting upon it. This contrasts with novice students, who often simplistically regard friction force as a resistive force (Caldas & Saltiel, 1995; Clement, 1993), and directly apply the equation  $f = \mu F_N$  to determine its magnitude for any and every friction force whether slipping is about to occur or not. (Arons, 1997; Boudreaux & Elby, 2020; Clement, 1993). In light of the preceding analysis, a profound understanding of the passivity of friction force is an important criterion to distinguish between experts and novices in dealing with problems related to friction force. Therefore, in this research, the passivity of friction force is chosen as the central idea.

**Figure 1**  
*Friction Force Conceptual Framework*



After thorough analysis and expert deliberation, the conceptual framework for friction force has been crafted and is illustrated in Figure 1. The central idea of this framework is the passivity of friction force, which integrates other related concepts within the subject matter. As previously stated, the inherent passivity of friction force necessitates that its effect within a given context is inferred from the actual state of the object's motion. Given other conditions are met (such as the contact surface is rough), the occurrence of relative motion between objects readily implies the presence of sliding friction. Conversely, when the object is relatively stationary, it is not possible to determine whether there is a tendency to relative motion directly by observing the object's motion state. So, it is impossible to determine whether there is static friction. Newton's laws of motion are the key link between force and motion, through Newton's laws of motion, the force on an object can be evaluated in terms of its motion (Arons, 1997; Ha et al., 2013). Hence, Newton's laws of motion are essential in analyzing friction force. Accordingly, the second tier of the conceptual framework encompasses Newton's laws of motion. The Newton's laws of motion discussed here encompass both predictive and interpretive methodologies for analyzing problems using these principles (Ha et al., 2013). The predictive approach allows for the forecasting of an object's subsequent motion when the quantitative characteristics of the forces acting upon it are known, such as their magnitude and direction. On the other hand, the interpretive method involves deducing the properties of the forces by observing the motion characteristics of the object when the forces' quantitative attributes are not precisely defined. Proficiency in Newton's laws of motion enhances the comprehension of friction force's passivity and aids in problem-solving (Ha et al., 2013).

The central idea is also interconnected with other concepts, such as the calculation equation for kinetic friction, reference of frame, kinematics, and force synthesis, which may collectively contribute to addressing problems related to friction force. The third tier involves intermediate reasoning processes and operational procedures, including force analysis, and mathematical and logical manipulation processes. For expert students, the third tier effectively connects with the concepts in the first and second tiers. However, novice students often apply these procedures as memorized routines for problem-solving, creating a disconnect from the central idea. The bottom tier comprises contextual elements, which commonly pertain to the problem's design attributes. These can be modified to generate a variety of contexts and task environments. For most problems related to friction force, the task goals typically involve several types: whether friction force is present, identifying its type, quantifying its magnitude, and determining its direction.

Furthermore, the arrows linking disparate components signify possible reasoning pathways for individuals ranging from novices to experts. Solid arrows depict the reasoning paths of experts, while dashed arrows represent possible paths for novices. Experts, utilizing the central idea, establish comprehensive connections among related concepts, reasoning processes, operational procedures, and contextual features within their knowledge structure. However, novices tend to combine contextual features with memorized operational procedures to constitute a disjointed knowledge structure. As illustrated in Figure 1, the linkages are marked with bidirectional arrows, signifying that an interaction can originate from either terminus. However, some students may possess unidirectional links between some connections, reflecting a fragmented knowledge structure. In summary, the conceptual framework integrates these levels and task objectives to visually represent the knowledge structures of students with varying levels of conceptual understanding.

### *Modeling Knowledge Structures of Students at Different Levels*

Employing the developed conceptual framework, this study stratifies students' comprehension of friction force across three levels: novice, intermediate, and expert:

- (1) Novice level: At this level, students exhibit fragmented knowledge structures, and establish only localized connections between contextual features of problems and memorized operational procedures. For instance, students can accurately calculate the magnitude of the friction force acting on an object moving on a horizontal surface using the formula  $f = \mu F_N = \mu mg$ . Yet, when dealing with an object on an inclined plane, they still apply the formula  $f = \mu mg$  to calculate (Besson et al., 2007). Furthermore, students misinterpret the effect of friction force (Caldas & Saltiel, 1995), which impedes relative motion, incorrectly as "hindering motion". This leads to errors in determining the direction of friction force, with students believing "the direction of friction force is opposite to the direction of object motion" (Prasitpong & Chitaree, 2010) or "the direction of friction force is opposite to the direction of external force" (Prasitpong & Chitaree, 2010). Students at this level lack a fundamental understanding of the passivity of friction force. Their comprehension of friction force is largely derived from everyday life experiences, and their reasoning pathway can be described as "contextual features → memorized equations → task goals".
- (2) Intermediate level: At this level, students possess a fundamental understanding of the passivity of friction force and acknowledge that friction force is not inherently a resistive force but can serve as a driving force contingent on the context. Consequently, when addressing problems, they evaluate the presence and effect of friction force based on the specific situation. These students have developed a more complex and interrelated knowledge structure, however, their lack of familiarity with the application of the Newtonian mechanical interpretive method hinders the complete integration of Newton's laws of motion within this framework. Therefore, their analysis of object motion is limited to determining the presence of relative motion. In situations where relative motion is not evident, they may face challenges in evaluating the likelihood of relative motion tendency between objects (Boudreaux & Elby, 2020). Furthermore, it is noteworthy that these students also tend to depend on memorization strategies, thus they excel in addressing problems within familiar settings but face challenges in reasoning and analysis when encountering unfamiliar contexts (Bao et al., 2002; Bao & Redish, 2006). For example, while proficient in analyzing the friction force acting on a pushed box, students demonstrate uncertainty when confronted with the analysis of the friction force experienced by an individual on a sled during skiing (Prasitpong & Chitaree, 2010). Therefore, their reasoning pathway can be described as "contextual features → the passivity of friction force → task goals".



- (3) Expert level: At this level, students possess a profound understanding of the passivity of friction force. Utilizing this as an anchor point, they integrate Newton's laws of motion with other related concepts and form a comprehensive and coherent knowledge structure. In tackling friction-related problems, these students first thoroughly observe the phenomenon. Then, they apply Newton's laws of motion to analyze the precise effects of friction, avoiding reliance on rote memorization (Ha et al., 2013).

Figure 2 illustrates an example problem, and the following discussion demonstrates the reasoning process of the expert based on this example. Experts possess a profound understanding of the passive nature of friction force and recognize that the friction acting on an object is contingent upon its state of motion as well as other active forces. Evidently, the pull is the active force. Consequently, their initial step is to analyze the motion state of the object. Due to the uncertainty regarding whether relative motion has occurred between objects A and B, it is not possible to ascertain whether the friction force is sliding or static. However, Newton's third law tells them that the friction force between objects A and B always occurs in pairs, and  $f_{ab} = -f_{ba}$ . Therefore, considering objects A and B as a system is a good approach to solving the problem. By applying Newton's second law to the system, they calculate the acceleration (the predictive methodology of Newton's laws). Following this, they proceed to analyze each object individually, again utilizing Newton's second law to determine the magnitude of the friction force (the interpretive methodology of Newton's laws). Guided by the central idea of the passivity of friction force, experts will not fall into the trap of directly applying the formula for sliding friction.

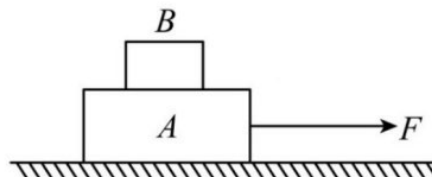
Their reasoning pathways can be characterized as "contextual features → passivity of friction force → Newton's laws of motion → task goals".

**Figure 2**  
Sample Question

#### Question

As shown in the figure,  $m_A = 2\text{ kg}$ ,  $m_B = 1\text{ kg}$ , A and B have a kinetic friction coefficient of 0.5, and the horizontal surface is smooth. When a 10 N horizontal force  $F$  is applied to pull A, what is the friction force between A and B?

- (A) 0 N
- (B) 5 N
- (C) 15 N
- (D)  $\frac{10}{3}$  N
- (E)  $\frac{20}{3}$  N



In summary, the conceptual framework delineates the knowledge structures of students across different levels of conceptual understanding, thereby explaining the challenges they face in understanding friction force. Furthermore, supplementary interviews are implemented to delve deeper into the students' reasoning pathways, thereby verifying the plausibility of the conceptual framework.

## Research Methodology

### Assessment Design

Drawing upon the conceptual framework for friction force, an evaluation instrument consisting of 14 multiple-choice items has been developed, designed to assess the level of students' knowledge integration. Some questions are adapted from previous research (Besson et al., 2007; Prasitpong & Chitaree, 2010), while the remainder are designed based on college entrance examination questions and practice problems.

Studies suggest that conceptual-framework-based assessments are particularly effective for evaluating both the integration of knowledge and the profundity of comprehension (Bao & Fritchman, 2021; Dai et al., 2019; Nie





et al., 2019; Zou et al., 2023). The assessments effectively leverage specific question design elements—such as contextual saliency and the connectedness of knowledge—that signify a level of knowledge integration.

Regarding the contextual saliency in question design, the assessment utilizes both typical and atypical questions, a strategy that has proven effective for developing assessments based on the conceptual framework (Bao & Fritchman, 2021; Dai et al., 2019; Zou et al., 2023). The contexts presented in typical questions are familiar to students and can be addressed using memorized solutions. In contrast, the context in atypical problems is unfamiliar to students, hence they need to synthesize the knowledge they have learned and make reasonable inferences to solve the problems. Novice and intermediate level students are capable of resolving problems within typical contexts utilizing memory-based strategies; however, they encounter difficulties when faced with atypical contexts. Conversely, experts demonstrate the ability to address problems across a wide range of contexts.

In terms of the connectedness of knowledge, this study draws upon previous theories (Biggs & Collis, 1982; Linn, 2006), and appropriately modifies them in accordance with the conceptual framework. The initial link types have been streamlined into three distinct levels: single-link, multi-link, and integrated-link. In single-link questions, students are tasked with establishing a singular association between discrete contextual components and concepts, which are typically solvable by recalling the relevant methodologies and operations. Multi-link problems require the establishment of associations among a broader range of contextual elements, multiple conceptual components, and operational steps; yet these associations are typically constrained in scope. Conversely, solving integrated-link problems requires a cohesive knowledge structure.

Expanding upon current methods for assessing knowledge integration, the test on friction force consists of 14 questions that are divided into three categories. Table 1 provides an overview of the types of questions and the expected problem-solving abilities of students at various stages of knowledge integration. The score patterns are denoted using the notation L/M/H, which corresponds to the levels of low, medium, and high performance, respectively. The scores are sequentially arranged to represent the progression from novice to intermediate and finally to expert proficiency levels. A specific sequence, such as LMH, is indicative of low scores attributed to novice participants, medium scores for those at the intermediate level, and consistently high scores for individuals at the expert proficiency level.

**Table 1**  
*Questions Type and Score Patterns*

Question Set	Question Type	Score Patterns
Set 1 (Q1, Q5, Q6, Q10, Q13)	Single-link typical	LHH
Set 2 (Q2, Q4, Q7, Q8)	Multi-link typical	LMH
Set 3 (Q3, Q9, Q11, Q12, Q14)	Integrated-link atypical	LLH

#### *Assessment Properties of the Test*

As shown in Table 2, the difficulty index is determined to be .575, and the discrimination index is calculated to be .527. According to DeVellis (1991), these values fall within an acceptable range. The reliability of the assessment instrument is evaluated using Cronbach's  $\alpha$  coefficient. The derived value of .77 exceeds the threshold of .65, indicating a reliable level of consistency within the test's measurements (DeVellis, 1991).

**Table 2**  
*Item Difficulty and Item Discrimination from the CTT Analysis*

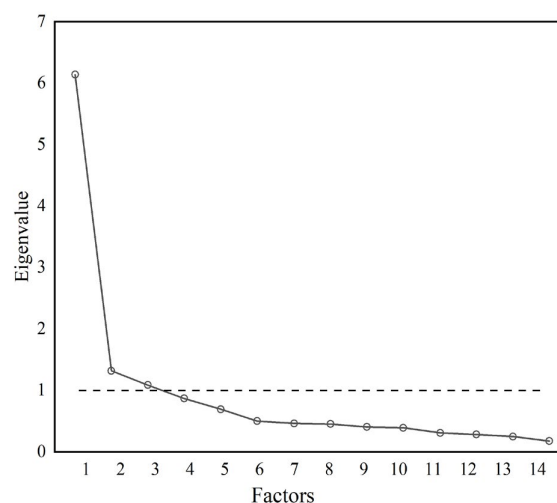
Question	Difficulty	Discrimination
Q1	.724	.444
Q2	.595	.642
Q3	.418	.487
Q4	.605	.588

Question	Difficulty	Discrimination
Q5	.716	.468
Q6	.609	.463
Q7	.503	.550
Q8	.670	.568
Q9	.568	.530
Q10	.686	.460
Q11	.441	.733
Q12	.488	.471
Q13	.682	.485
Q14	.347	.486
Average	.575	.527

The validity of the test was evaluated from both content validity and construct validity. In the development of the assessment instrument, in addition to the author, several high school physics teachers, educational researchers, and three graduate students specializing in physics within the team were collaboratively involved in the compilation process. By encompassing multiple pilot tests and feedback from faculty and graduate students at the author's research institution, the design underwent a meticulous development and refinement process. The final version of the test content was unanimously deemed scientifically accurate and effective by the design and evaluation team; therefore, the test has good content validity.

Exploratory factor analysis (EFA) was performed to evaluate the construct validity of the test. As shown by the scree plot in Figure 3, there are three factors with eigenvalues greater than one, and they can explain 59% of the variance (with factors 1, 2, and 3 accounting for 23%, 19%, and 17%, respectively). Therefore, the student's performance on this test can be fully explained by three factors. Table 3 presents the factor loadings for each item, where a factor loading represents the degree of correlation between an item and its underlying factor (loadings below 0.35 have been omitted). It is evident that items Q1, Q5, Q6, Q10, and Q13 exhibit substantial and significant associations with Factor 1. Items Q2, Q4, Q7, and Q8 are closely related to Factor 2, while items Q3, Q9, Q11, Q12, and Q14 are closely associated with Factor 3. This distribution is entirely consistent with the pre-established subject categories, thereby indicating that the test possesses strong construct validity.

**Figure 3**  
Scree Plot of EFA



**Table 3***Pattern Matrix*

Item	Factor 1	Factor 2	Factor 3
Q13	.80		
Q1	.76		
Q10	.76		
Q6	.75		
Q5	.35		
Q4		.94	
Q7		.89	
Q2		.83	
Q8		.38	
Q11			.95
Q12			.81
Q9			.81
Q3			.73
Q14			.68

*Participants*

This study involved 598 grade-10 students from 12 classes in an upper-secondary school in China, comprising 317 female and 281 male students, with ages ranging from 15 to 16 years old. Among them, four classes were designated as “better classes”, where students exhibited superior performance in physics, while the remaining eight were considered ordinary classes with average physics achievements. This stratification ensured a diverse range of abilities among the participants in the study. All participants had been exposed to the physics curriculum, specifically covering topics on friction force and Newton’s laws of motion, which is a prerequisite for the study. Prior to the study, all participants were briefed on its objectives and the voluntary nature of their involvement. They were assured that their participation would not affect their physics grades, and the test results would be used independently.

*Data Collection and Analysis*

The assessment was conducted in December 2023. The duration of the assessment was 40 minutes. Subsequent to the assessment, a voluntary sample of 30 students from the participant pool was selected for interviews. These interviews were conducted within one-week post-assessment, with each session lasting approximately 30 minutes.

After the test, the students’ understanding of the concept of friction was explored and verified by quantitative and qualitative methods. Previous research has shown that students’ level of knowledge integration can be revealed in the score gap of questions with different link types and context saliency (Xie et al., 2021; Xu et al., 2020; Zou et al., 2023). Therefore, quantitative analysis focuses on identifying differences in students’ performance in various problem designs, thereby inferring students’ knowledge structure. The statistical significance of the comparative analysis between results derived from different question designs is ascertained through the application of one-way analysis of variance (ANOVA). Subsequent to this, the analysis was deepened by employing the paired samples *t*-test to examine pairwise group differences, complemented by Cohen’s *d* to estimate the effect size. The qualitative analysis is further used to analyze the knowledge structures of students with varying levels of knowledge integration of friction force.



## Research Results

### Results of the Quantitative Study

The scores of students on questions categorized by varying question types are detailed in Table 4. The results show that students performed best in single-link typical questions and performed worst in integrated-link atypical questions, which aligns very well with our previous expectations. A one-way analysis of variance (ANOVA) revealed statistically significant differences among the scores for the three question sets [ $F(2, 1791) = 186.644, p < .001$ ]. The distinctions were more explicitly revealed through the application of paired  $t$ -test [ $t_{(SM)}(597) = 8.407, p < .001, d = .344$ ;  $t_{(SI)}(597) = 24.545, p < .001, d = 1.004$ ;  $t_{(MI)}(597) = 17.699, p < .001, d = .724$ ]. In summary, the result suggests that questions designed with varying link types and context saliency are effective in assessing the level of students' knowledge integration.

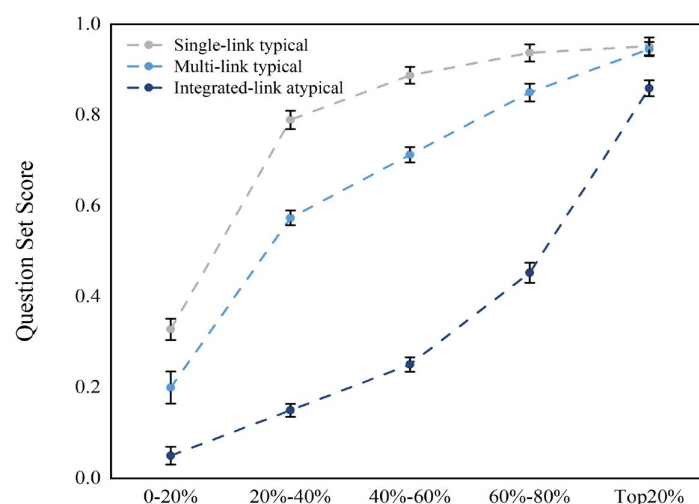
**Table 4**  
*Students' Scores on Different Question Sets*

Question Set	Link type	Context type	<i>M</i>	<i>SE</i>
Set 1	Single-link	Typical	.773	.010
Set 2	Multi-link	Typical	.656	.012
Set 3	Integrated-link	Atypical	.361	.012
Total Score			.591	.011

Notes: *SE* = Standard Error; *M* = Mean

This assessment served two purposes: first, to validate the effectiveness of the friction force conceptual framework; and second, to infer students' levels of knowledge integration by analyzing their performance across different question sets. To achieve these objectives, students were initially categorized into five groups based on their total test scores, with each group representing 20% of the sample. Subsequently, the study compared the scores of each group on different types of questions, as illustrated in Figure 4.

**Figure 4**  
*Scores on Different Types of Questions by Students of Different Levels (With Error Bars Denoting Standard Error)*



As shown in Figure 4, students in the lower group (0-20%) perform consistently poorly across all question designs. These students can only address a subset of the single-link typical questions, yet they struggle with the

majority of multi-link typical and integrated-link atypical questions. Consequently, this suggests that they have not grasped the fundamental concept and remain at an early stage in the process of knowledge integration. Intermediate students (20%-80%) are able to solve most single-link typical and multi-link typical questions but perform poorly on integrated-link atypical questions. This implies that these students have acquired a rudimentary comprehension of the concept of friction force, yet they have not fully developed an integrated knowledge structure. The findings indicate that these students have attained a moderate degree of knowledge integration. They excel at employing memorization strategies to address questions in familiar contexts but struggle with the flexible transfer and application of knowledge to unfamiliar contexts. Ultimately, students in the advanced group (80%-100%) demonstrate nearly equivalent performance across these three types of questions and achieve relatively high scores in each category. This indicates that these students have profoundly understood the central idea and integrated it with other related concepts to construct a comprehensive knowledge structure. Thereby, they have achieved a level of expert.

Furthermore, by observing the slope change of the curve, it can be seen that the students' understanding of friction force goes through three different stages. The initial stage (0-40%) is defined by the aggregation of information. At this stage, students gather a variety of contextual variables and associated concepts into their knowledge structures. This process is guided by the problem's apparent characteristics. Consequently, their ability to solve problems in familiar contexts (including single-link typical questions and multi-link typical questions) is growing at almost the same rate, while their ability to tackle atypical problems improves at a slower pace. The second stage (40%-80%) involves the construction of connections between concepts. At this stage, students have gradually gathered sufficient key information and fundamental concepts, and they begin to establish connections between them. As a result, the gap between students' performance in solving multi-link typical and single-link typical problems gradually decreases. The third stage (Top 20%) is the comprehensive integration of the knowledge structure. Here, students continuously build connections between concepts and variables that are centered on the central concept. This process ultimately leads to the formation of an expert-level understanding of the concept. As a result, the gap in the performance of students at this level on all types of questions gradually decreases and eventually converges.

The analysis reveals that the evolution of students' conceptual understanding aligns with the process of knowledge integration. That is, students continuously learn new knowledge and gradually connect it with the central idea, eventually forming an integrated knowledge structure. This facilitates the transition from novice to expert and achieves a deep understanding of the concept.

### *Results of the Qualitative Study*

Within one week of the student taking the test, a think-out-loud interview was conducted with 30 individuals from the test group, including novice (10), intermediate (10), and expert (10) students. During the interviews, students were instructed to provide answers to questions along with their reasoning processes.

**Novice level:** In the problem-solving process, these students generally associate the context of the problem with operational steps and equations they have memorized to derive a solution. Moreover, students at this proficiency level frequently exhibit a deficient grasp of the core concept, as illustrated by the subsequent interview excerpts:

*Student A: (Answered B to Q1) "The object in the problem moves to the right. Friction force hinders the motion of an object, which means that the direction of friction force is opposite to the motion of the object, so the direction of friction force is to the left." (Answered A to Q4) "Similar to the previous analysis, the object moves to the right, so, the friction force is directed to the left."*

*Student B: (Answered A to Q10) "Because the masses of the three objects are the same, and the friction coefficient with the ground is the same, we can determine that the friction force on the three objects is equal using the equation  $f = \mu F_N$ ."*

*Student C: (Answered A to Q11) "The context described in this question is commonly encountered in daily life. When writing with a brush moving to the right, the paper tends to slide to the right. Placing an object on the left side of the paper prevents sliding. Therefore, the object exerts friction force on the paper, directed to the left."*

Q1 represents a quintessential example, one that students have previously encountered within analogous

problem scenarios. Thus, recalling the outcomes of previous questions enables them to provide the correct response. However, when explaining the reasoning, student A referred to the effect of friction force as resisting motion instead of resisting relative motion, indicating an incorrect grasp of the central idea. This misunderstanding is reflected in their inability to successfully answer Q4, which is a multi-link question. Similarly, student C, like student A, did not correctly understand the essence of friction force. Consequently, when attempting to solve question Q11, which involves integrated concepts, student C had to rely on intuition to guess the answer. Furthermore, student B's response reveals that novice students can memorize equations but do not grasp their meaning, leading to a misunderstanding of the normal force  $F_N$  as the gravitational force of the object when applying the formula  $f = \mu F_N$ .

In summary, novice students' understanding of friction force is superficial, lacking a thorough comprehension of its essence. Most of their perception comes from life experience, for instance, they commonly perceive friction force as a resistive force, which always acts opposite to the direction of motion. The interview results corroborate Table 4, indicating that novice students can successfully solve some single-link typical questions but struggle with multi-link typical and integrated-link atypical questions.

**Intermediate Level:** These students have developed an appreciation for the passivity inherent in friction force, moving beyond the rudimentary perception of friction as solely a force of resistance. This enhanced understanding is exemplified in the forthcoming interview excerpts:

*Student D: (Answered B to Q1 and Q4) "In the first question, the object is moving to the right relative to the ground. Based on the effects of friction force, which impedes the relative motion between objects, so the object experiences friction force from the ground to its left. In the fourth question, when the object is in segment AC, its velocity is less than that of the conveyor belt, so the object moves to the left relative to the conveyor belt, experiencing friction force from the conveyor belt to its right."*

It is evident that intermediate-level students possess a foundational grasp of the concept of frictional passivity and are able to solve most single-link and multi-link questions. However, their knowledge structure remains incompletely synthesized, which often leads to challenges in addressing problems of integrated link. This issue is exemplified by the interview excerpts presented below:

*Student E: (Answered E to Q3) "There is no relative motion between people and objects, so there is no friction force between them." (Follow-up question: What are the conditions for the existence of friction force?) "It is mentioned in the textbook, there are three conditions. First, rough contact surfaces; second, compression between objects; third, relative motion or relative motion trends between objects." (Follow-up question: Can we conclude that there is no friction force if there is no relative motion between them? Is there a relative motion trend between them?) "Probably not, because the person is stationary relative to the escalator, and I'm not quite sure either."*

*Student F: (Answered A to Q6) "Initially, consider the diagram depicted on the left. The foot exerts a force in a posterior direction, resulting in a relative retrograde motion trend of the foot against the ground. Consequently, the friction force acting on the foot is directed anteriorly. Similarly, as depicted in the diagram on the right, when the foot exerts a force in an anterior direction, it exhibits a forward motion trend relative to the ground, and thus, the frictional force on the foot is directed posteriorly."*

Analysis of the above interviews shows that in the absence of relative sliding between objects, intermediate-level students rely primarily on intuition or conjecture to assess whether there is static friction between objects, rather than applying Newton's laws of motion. Subsequent interviews show that while these students had mastered Newton's laws of motion, they had not yet incorporated these principles into their knowledge structure of friction force.

*Student G: (Answered A to Q11) "When writing to the right, the brush moves to the right relative to the paper, so the brush is subjected to the leftward friction given to it by the paper. And according to Newton's third law, the brush exerts a rightward friction force on the paper. But the paper remains stationary, so the paper has a tendency to move to the right with respect to the object, so the paper is subjected to the leftward friction exerted on it by the object." (Follow-up question: Does that object experience the friction force of the paper against it?) "According to Newton's third law, if an object has a left-facing friction against the paper, then the paper has a right-facing friction against*

*the object. But in this case, the object has a net force to the right in the horizontal direction, and according to Newton's laws of motion, it should move to the right. Oh, I was wrong in my analysis, there is no friction between the object and the paper."*

Newtonian mechanics can be approached through two distinct methodologies: the predictive method and the explanatory method. Interviews have revealed that intermediate level students demonstrate proficiency in utilizing the predictive method, adeptly deducing the motion states of objects by examining the forces exerted upon them. Conversely, the application of the explanatory method presents a more formidable challenge for these students. While a thorough comprehension of force dynamics enables accurate predictions of an object's trajectory, the analysis of friction force introduces complexities. Friction force, not being a fundamental force governed by universal laws, necessitates the complementary use of the explanatory method alongside the predictive approach.

In summary, intermediate-level students have gained a preliminary understanding of the passivity of friction force and recognized the importance of analyzing the effects of friction force based on the motion state of objects. However, intermediate-level students still fall short in their comprehension of friction force, due to difficulties in integrating Newton's laws of motion into their knowledge structure of friction force. This leads to challenges in determining the relative motion tendencies between objects.

**Expert Level:** Students at the expert level are capable of solving almost all questions. Particularly, students at this level seem to have applied their knowledge of Newton's laws of motion to analyze questions involving friction force, as evident from the interview excerpts below:

*Student H: (Answered D to Q3) "Firstly, analyze the motion state of the person; there is an acceleration directed towards the upper right corner. This acceleration can be decomposed into vertical and horizontal components. Consequently, the net force in the vertical direction is upward, indicating a state of apparent weight increase. In the horizontal direction, the net force is to the right, and since there is no external force acting horizontally, there must be a friction force acting to the right. As the person is stationary relative to the escalator, this friction is static friction."*

*Student I: (Answered A to Q9) "Firstly, analyze object B, which definitely experiences friction force from the slope because it meets the conditions for friction force to occur. Therefore, the acceleration of object B along the slope is less than the component of gravitational acceleration along the slope. Next, analyze object A. Since object A is relatively stationary with object B, their velocities are consistent at all times, and thus their accelerations are the same. Suppose object A does not experience friction force from object B; then its acceleration along the slope would definitely be greater than that of object B, making this assumption unreasonable. Therefore, object A does experience friction force from object B, and the direction of this friction is upward along the contact surface."*

*Student J: (Answered C to Q11) "During the process of moving the brush to the right, the brush moves to the right relative to the paper. Therefore, the paper exerts leftward friction force on the brush. According to Newton's third law, the brush exerts rightward friction on the paper. Since the paper is stationary, the table applies leftward friction force on the paper." (Follow-up question: Why can't it be the object providing leftward friction to the paper?) "Because if the object imparts leftward friction to the paper, according to Newton's third law, the paper must exert rightward force on the object. According to Newton's first law, the object would change its state of rest and move to the right. However, the object remains stationary, which implies from Newton's first law that there is no friction force between the object and the paper."*

The reasoning processes of these three students reveal a deep understanding of the passivity of friction force. When addressing problems, they determine the actual effects of friction force by analyzing the motion state of objects. In situations where there is no relative motion between objects, they can apply Newton's laws of motion to assess the tendency of relative motion, and thereby ascertain the existence and specific effects of static friction. It is particularly notable that expert-level students have a bidirectional connection between the central idea and Newton's laws of motion. This is reflected in their ability to use these laws to analyze the effects of friction force during problem-solving and to subsequently validate the accuracy of their prior reasoning after the problem has been resolved.

In essence, students at the expert level have cultivated an advanced and cohesive cognitive framework, which empowers them to adeptly utilize pertinent knowledge to address problems across diverse contexts.



## Discussion

By employing quantitative and qualitative methods, this study empirically verifies the differences in knowledge structures among students with varying levels of conceptual understanding. These findings are consistent with previous research (Bao & Fritchman, 2021; Dai et al., 2019; Linn, 2006; Zou et al., 2023). Furthermore, compared to previous studies diagnosing misconceptions about friction force (Ha et al., 2013; Kızılcık et al., 2021), this study provides a deeper explanation for the origins of these difficulties. The study identifies three levels of students' understanding of friction: novice, intermediate, and expert. By analyzing their reasoning pathways during problem-solving, the origins of their misconceptions can be dissected.

**Novice Level:** At this level, students' knowledge structures are fragmented. When addressing problems, their primary focus is on the contextual elements, and they depend on their recollection of equations to facilitate the matching process. Although they can solve some single-link typical questions using equations or procedural operations from memory, they often encounter difficulties with multi-link typical and integrated-link atypical questions. The students' understanding of friction is confined to their everyday experiences, leading them to select answers based on experience rather than solving problems through analytical reasoning. For example, they often simplistically regard friction force as a resistive force (Caldas & Saltiel, 1995; Clement, 1993), and directly apply the equation  $f = \mu F_N$  to determine its magnitude (Arons, 1997; Boudreaux & Elby, 2020; Clement, 1993).

**Intermediate Level:** Intermediate-level students begin to grasp the passivity of friction force and recognize the importance of analyzing the effect of friction force based on the objects' motion state, rather than merely defining it as a resistive force like novice students. However, because the explanatory method of Newton's laws of motion is not mastered, they cannot adequately establish a connection between force and motion. They can judge the motion of an object by the force on it, and vice versa (Boudreaux & Elby, 2020; Ha et al., 2013). Consequently, these students have failed to integrate Newton's laws of motion into their knowledge structure under the topic of friction force. For example, in an unfamiliar problem situation, they cannot determine whether there is a tendency for objects to move relative to each other (Ha et al., 2013). Students at this level continue to employ memorization strategies to solve problems, excelling at addressing questions in familiar contexts but often at a loss when confronted with unfamiliar contexts.

**Expert Level:** Expert-level students integrate Newton's laws of motion into their knowledge structure on the topic of friction force, thereby establishing a bridge between force and motion, and thus gaining a deeper understanding of the passivity inherent in friction. They are capable of solving almost all problems and consistently provide reasonable explanations. These students do not rely on memorized equations and are able to apply the central idea consistently to solve problems, regardless of whether the context is familiar or unfamiliar (Arons, 1997; Ha et al., 2013).

Additionally, by comparing the scientific reasoning processes of students with different levels of conceptual understanding while solving problems, the progression from superficial to deep understanding is revealed: from the accumulation of surface-level contextual information to the construction of more complex knowledge structures, and finally to fully integrated conceptual understanding (Bao et al., 2002; Biggs & Collis, 1982). This study found that problem situations play a crucial role in this process. Familiar problem contexts can provide a "scaffolding" for students to acquire new knowledge, guiding them to start from their pre-existing understanding and integrate it with scientific concepts. This is what Piaget (1976) described as the "assimilation" process, which represents the "quantitative change" phase in the learning of knowledge. Conversely, unfamiliar problem contexts can offer further opportunities and points of leverage for the integration of knowledge, encouraging students to reflect on their personal experiences and fragmented knowledge from a scientific and systematic perspective and to modify and reorganize their existing knowledge structures. This is the "accommodation" (Piaget, 1976) process, signifying the "qualitative change" phase in knowledge acquisition. Hence, the intentional arrangement and meticulous design of problem contexts are essential for students to both acquire new knowledge and integrate existing knowledge. The design of unfamiliar contexts aims to enhance the integration and deep comprehension of students' knowledge, discouraging over-reliance on familiar contexts and the use of rote learning strategies that lead to superficial learning.

Moreover, the results indicate that the central idea plays a pivotal role in the integration of knowledge within a specific thematic context. Through analyzing the students' performance in the tests and interviews, we have discerned that within the theme of friction force, the passivity of friction plays a pivotal role, reflects its characteristics and serves as an anchor to connect other related concepts. Our research findings indicate that the majority of students have not developed a profound understanding of friction force, but rely on extensive problem-solving practice to become proficient at answering questions in familiar contexts. However, when faced with unfamiliar scenarios, they lack the capability to solve problems. Only a minority of students have a deep understanding of the passivity of friction, thereby forming a highly integrated knowledge structure that enables them to flexibly address a variety





of problems. Therefore, teachers should tailor their instruction according to the characteristics of students at different levels of knowledge integration. For instance, with novice-level students, teachers should help them overcome misconceptions about friction force, such as the misconception that friction force is a resistive force. Teachers can provide scenarios where friction force acts as a driving force to induce cognitive conflict, thereby facilitating conceptual change. For intermediate-level students, instruction should assist in the integration of the central idea with Newton's laws of motion. To achieve this, teachers should help students understand the use of Newtonian mechanics explanatory methods (Ha et al., 2013).

## Conclusions and Implications

This study has constructed a conceptual framework for friction force, assessing students' levels of understanding from the perspective of knowledge integration. The research indicates that students' comprehension of friction force can be categorized into three levels: novice, intermediate, and expert. Students at different levels exhibit varied reasoning paths when solving problems related to friction force, which also elucidates the reasons behind the emergence of various misconceptions during the learning process of friction force. Furthermore, the passivity of friction force is identified as a pivotal element in forming an integrated knowledge structure, which is essential for achieving profound conceptual understanding. The study provides valuable insights for educators in the development of targeted curricula, aimed at enhancing students' deep comprehension. The findings suggest that constructing a conceptual framework is effective for modeling the knowledge structure of students at different levels of conceptual understanding. Utilizing this framework, educators can identify gaps in students' knowledge structures and implement targeted instruction to address the specific needs of students at various stages. Furthermore, the central idea plays a pivotal role in the knowledge structure, and it is recommended that educators emphasize its importance when designing curricula. For example, when teaching about friction force, the focus should be on understanding its passivity. This study also has several limitations that warrant attention. Firstly, the research focused solely on static and sliding friction force, excluding rolling friction force, thereby necessitating the development and validation of a more comprehensive conceptual framework to fully assess students' understanding of friction force. Secondly, the sample was exclusively composed of students from China, which may restrict the generalizability of the findings. Future research should aim to broaden the sample to include a more diverse demographic, in order to substantiate the applicability of the conceptual framework across different cultural and educational contexts.

## Declaration of Interest

The authors declare no conflict of interest.

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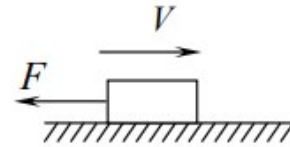


**Appendix***Friction Force Test*

For the following questions, you may use a value of  $g = 10 \text{ m/s}^2$  as the approximate value for the acceleration due to gravity.

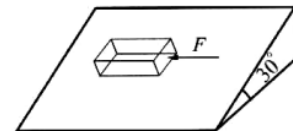
1. As shown in the figure, an object with a mass of 10 kg is moving to the right on a horizontal surface with a kinetic friction coefficient of 0.1, during its motion, it is subjected to a horizontal force  $F$  to the left with a magnitude of 20 N. Regarding the magnitude and direction of friction force acting on the object, which of the following statements is true?

- (A) 10 N, to the right
- (B) 10 N, to the left
- (C) 20 N, to the right
- (D) 20 N, to the left



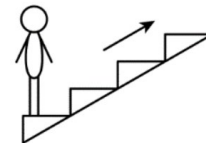
2. As shown in the figure, a block of mass 0.8 kg is at rest on an inclined plane with an angle of  $30^\circ$ . If a horizontal force of 3 N, parallel to the inclined plane, is applied to push the block, and the block remains stationary, what is the magnitude of the friction force acting on the block?

- (A) 3 N
- (B) 4 N
- (C) 5 N
- (D) 8 N



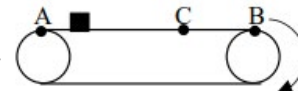
3. As shown in the figure, Xiao Ming is riding an escalator upward, and the escalator is accelerating upward. Xiao Ming is stationary relative to the escalator. What is the direction of the friction force acting on Xiao Ming?

- (A) Opposite to the direction of motion.
- (B) Same as the direction of motion.
- (C) Horizontally to the left.
- (D) Horizontally to the right.
- (E) No friction force is acting.



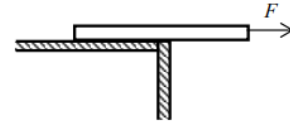
4. As shown in the figure, a conveyor device is placed horizontally with a constant rotational speed. An object is gently placed at position A on the conveyor belt, and when position C is reached, the object has the same velocity as the conveyor belt. During the conveyance, what is the situation regarding the friction force acting on the object?

- (A) The object experiences sliding friction in the AC section, directed horizontally to the left.
- (B) The object experiences sliding friction in the AC section, directed horizontally to the right.
- (C) The object experiences static friction in the CB section, directed horizontally to the left.
- (D) The object experiences static friction in the CB section, directed horizontally to the right.



5. As shown in the figure, a horizontal force  $F$  of 20 N is applied to the right to pull a wooden board, causing it to move to the right. The mass of the board is 10 kg, and the kinetic friction coefficient between the board and the table surface is 0.1. What is the magnitude of the friction force acting on the board before it tips over?

- (A) 10 N  
(B) 20 N  
(C) Decreasing from 20 N to 0  
(D) Decreasing from 10 N to 0



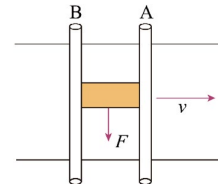
6. During the two moments of Xiao Ming's run, as depicted in the diagrams,  $f_1$  and  $f_2$  denote the friction forces acting on Xiao Ming in the first and second moments, respectively. Regarding the direction of  $f_1$  and  $f_2$ , the accurate statement is:

- (A)  $f_1$  forward,  $f_2$  backward  
(B)  $f_1$  forward,  $f_2$  forward  
(C)  $f_1$  backward,  $f_2$  backward  
(D)  $f_1$  backward,  $f_2$  forward



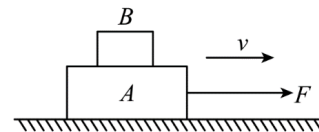
7. As shown in the figure, the object is placed on a horizontal conveyor belt, and fixed, smooth horizontal rods A and B are installed on both sides of the object, which restrict the movement of the object only in between. The mass of the object is  $m$ , the kinetic friction coefficient between the object and the conveyor belt is  $\mu$ , and the object moves at a constant speed  $v_0$ . The conveyor belt moves to the right, its speed can be changed. Regarding the friction force  $f$  experienced by the object, which of the following statements is true?

- (A)  $f = F$   
(B)  $f = \mu mg$   
(C) The direction of  $f$  is opposite to that of  $F$   
(D) The direction of  $f$  is horizontally to the left.



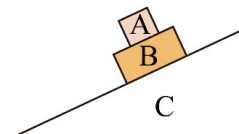
8. As shown in the figure, two blocks A and B are stacked together and move to the right on a rough horizontal surface under the action of an external force  $F$ . If A and B remain relatively stationary during the motion, which of the following statements is true about the friction force  $f$  experienced by block B?

- (A) If A and B move together at a constant speed,  $f$  is directed to the left.  
(B) If A and B move together at a constant speed,  $f$  is directed to the right.  
(C) If A and B move together with acceleration,  $f$  is directed to the left.  
(D) If A and B move together with deceleration,  $f$  is directed to the left.



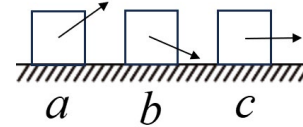
9. As shown in the figure, on a fixed inclined plane, there are objects A and B, and all contact surfaces are rough. Objects A and B are relatively stationary and accelerate down the slope together. Regarding the friction force  $f$  experienced by object A, which of the following statements is true?

- (A)  $f$  is directed upward along the contact surface.  
(B)  $f$  is directed downward along the contact surface.  
(C) There is no friction force between object B and object A.  
(D) The motion trend of object A relative to object B is uncertain.



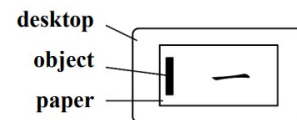
10. As shown in the figure, objects **a**, **b**, and **c**, which are identical, are moving on the ground under the force  $F$ . Assuming the magnitude of  $F$  is the same in all three cases. Regarding the friction force experienced by the object, which of the following statements is true?

- (A) All three are equal in magnitude
- (B) Object A experiences the greatest
- (C) Object B experiences the greatest
- (D) Object C experiences the greatest



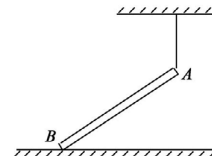
11. As shown in the figure, Xiao Ming is writing on a horizontal desktop. To prevent slipping, an object is placed near the edge of the white paper on the left side. When writing to the right, both the paperweight and the paper remain stationary relative to the desktop. Which of the following statements is true?

- (A) The object exerts a friction force on the paper, directed to the left.
- (B) The object exerts a friction force on the paper, directed to the right.
- (C) The desktop exerts a friction force on the paper, directed to the left.
- (D) The pen exerts a friction force on the paper, directed to the left.



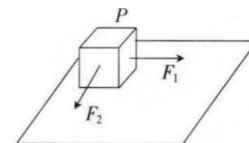
12. As shown in the figure, a worker suspends a steel pipe between the ceiling and the horizontal ground using a lightweight rope. When the pipe is stationary, the rope is vertical. Which of the following statements is true?

- (A) The ground exerts a friction force on the pipe, directed to the right.
- (B) The ground exerts a friction force on the pipe, directed to the left.
- (C) There is no friction force between the ground and the pipe.
- (D) The motion trend of the pipe relative to the ground is uncertain.



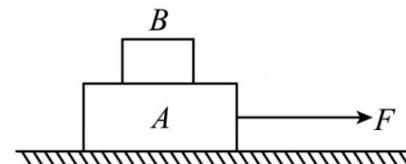
13. As shown in the figure, block P is stationary on a horizontal plane. When two mutually perpendicular horizontal forces,  $F_1$  and  $F_2$  ( $F_1 > F_2$ ), are applied to it individually, the block moves in the direction of  $F_1$  and  $F_2$ , with the respective frictional forces being  $f_1$  and  $f_2$ . If both of these horizontal forces are applied at the same time from an initial state of rest, the magnitude of the sliding friction force acting on the block is  $f_3$ . The accurate statement about the magnitudes of these three friction forces is:

- (A)  $f_1 = f_2 < f_3$
- (B)  $f_1 = f_2 = f_3$
- (C)  $f_1 > f_2 = f_3$
- (D)  $f_1 < f_2 < f_3$



14. As shown in the figure,  $m_A = 2\text{ kg}$ ,  $m_B = 1\text{ kg}$ , A and B have a kinetic friction coefficient of 0.5, and the horizontal surface is smooth. When a 10 N horizontal force  $F$  is applied to pull A, what is the friction force between A and B?

- (A) 0 N
- (B) 5 N
- (C) 15 N
- (D)  $\frac{10}{3}\text{ N}$
- (E)  $\frac{20}{3}\text{ N}$



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